# Edexcel GCE

## Further Pure Mathematics FP2
## Advanced Level

### Friday 22 June 2007 − Afternoon
### Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP2), the paper reference (6675), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**1.** Evaluate $\displaystyle\int_{1}^{3} \frac{1}{\sqrt{(x^2+4x-5)}}\,dx$, giving your answer as an exact logarithm.

**(5)**

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**2.** The ellipse $D$ has equation $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ and the ellipse $E$ has equation $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

(*a*) Sketch $D$ and $E$ on the same diagram, showing the coordinates of the points where each curve crosses the axes.

**(3)**

The point $S$ is a focus of $D$ and the point $T$ is a focus of $E$.

(*b*) Find the length of $ST$.

**(5)**

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**3.** The curve $C$ has equation
$$y = \tfrac{1}{4}(2x^2 - \ln x), \quad x > 0.$$

Find the length of $C$ from $x = 0.5$ to $x = 2$, giving your answer in the form $a + b\ln 2$, where $a$ and $b$ are rational numbers.

**(7)**

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**4.** (*a*) Starting from the definitions of cosh and sinh in terms of exponentials, prove that
$$\cosh(A - B) = \cosh A \cosh B - \sinh A \sinh B.$$

**(3)**

(*b*) Hence, or otherwise, given that $\cosh(x - 1) = \sinh x$, show that
$$\tanh x = \frac{e^2 + 1}{e^2 + 2e - 1}.$$

**(4)**

**5.** The curve $C$ has parametric equations

$$x = t - \sin 2t, \quad y = \cos 2t, \quad 0 \le t < \pi.$$

(a) Find, in terms of $\cos 2t$ only, an expression for the radius of curvature of $C$.

**(6)**

(b) Write down the least value of $y$ and hence find the radius of curvature of $C$ at the point where $y$ has this least value.

**(2)**

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**6.** Given that $I_n = \displaystyle\int_0^8 x^n (8-x)^{\frac{1}{3}} \ \mathrm{d}x, \quad n \ge 0,$

(a) show that $I_n = \dfrac{24n}{3n+4} I_{n-1}, \quad n \ge 1.$

**(6)**

(b) Hence find the exact value of $\displaystyle\int_0^8 x(x+5)(8-x)^{\frac{1}{3}} \ \mathrm{d}x.$
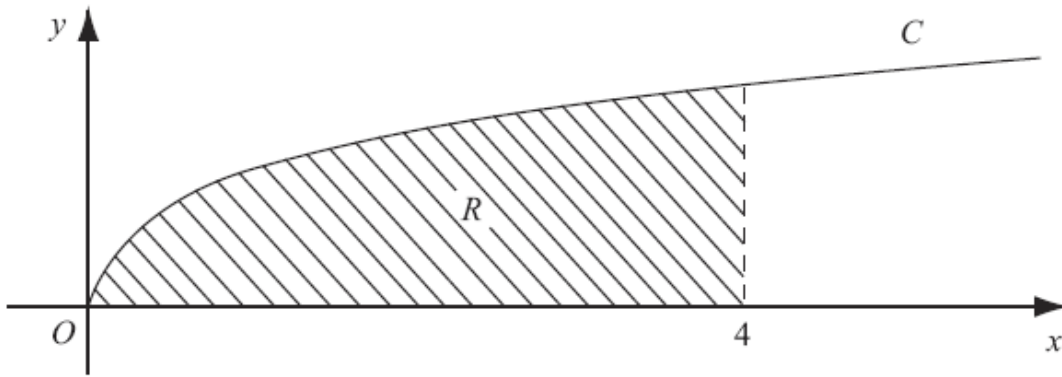
**(6)**

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**7.**



**Figure 1**

Figure 1 shows part of the curve $C$ with equation $y = \operatorname{arsinh}(\sqrt{x})$, $x \geq 0$.

(*a*) Find the gradient of $C$ at the point where $x = 4$.

(3)

The region $R$, shown shaded in Figure 1, is bounded by $C$, the $x$-axis and the line $x = 4$.

(*b*) Using the substitution $x = \sinh^2 \theta$, or otherwise, show that the area of $R$ is

$$k \ln(2 + \sqrt{5}) - \sqrt{5},$$

where $k$ is a constant to be found.

(10)

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**8.** The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, $p \neq q$, lie on the parabola $C$ with equation $y^2 = 4ax$, where $a$ is a constant.

(*a*) Show that an equation for the chord $PQ$ is $(p + q)\, y = 2(x + apq)$.

(3)

The normals to $C$ at $P$ and $Q$ meet at the point $R$.

(*b*) Show that the coordinates of $R$ are $(a(p^2 + q^2 + pq + 2),\ -apq(p + q))$.

(7)

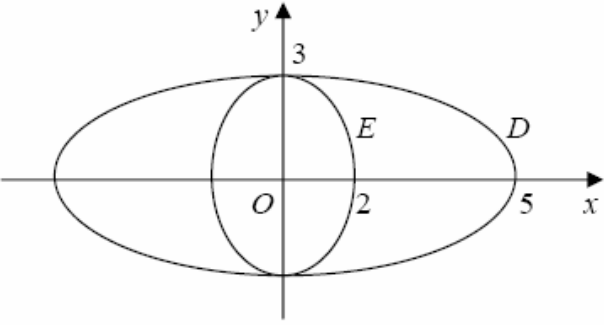Given that the points $P$ and $Q$ vary such that $PQ$ always passes through the point $(5a, 0)$,

(*c*) find, in the form $y^2 = f(x)$, an equation for the locus of $R$.

(5)

**TOTAL FOR PAPER: 75 MARKS**

**END**

| Question Number | Scheme | Marks |
|---|---|---|
| 1. | $$x^2 + 4x - 5 = (x+2)^2 - 9$$ | B1 |
| | $$\int \frac{1}{\sqrt{((x+2)^2 - 9)}}\, dx = \operatorname{arcosh} \frac{x+2}{3}$$ | M1 A1ft |
| | ft their completing the square, requires arcosh | |
| | $$\left[ \operatorname{arcosh} \frac{x+2}{3} \right]_1^3 = \operatorname{arcosh} \frac{5}{3}\ (-\operatorname{arcosh} 1)$$ | |
| | $$= \ln\left( \frac{5}{3} + \sqrt{\frac{25}{9} - 1} \right) = \ln\left( \frac{5}{3} + \frac{4}{3} \right) = \ln 3$$ | M1 A1 (5) |
| | | [5] |
| | *Alternative* | |
| | $$x^2 + 4x - 5 = (x+2)^2 - 9$$ | B1 |
| | Let $x + 2 = 3\sec\theta$, $\qquad \dfrac{dx}{d\theta} = 3\sec\theta\tan\theta$ | |
| | $$\int \frac{1}{\sqrt{((x+2)^2 - 9)}}\, dx = \int \frac{3\sec\theta\tan\theta}{\sqrt{(9\sec^2\theta - 9)}}\, d\theta$$ | M1 |
| | $$= \int \sec\theta\, d\theta$$ | A1ft |
| | $$\left[ \ln(\sec\theta + \tan\theta) \right]_{\operatorname{arcsec}1}^{\operatorname{arcsec}\frac{5}{3}} = \ln\left( \frac{5}{3} + \frac{4}{3} \right) = \ln 3$$ | M1 A1 (5) |

| Question Number | Scheme | Marks |
|---|---|---|
| 2. | (a)  | |

One ellipse centred at $O$ — B1

Another ellipse, centred at $O$, touching on $y$-axis — B1

Intersections: At least 2, 5, and 3 shown correctly — B1   (3)

(b)  Using $b^2 = a^2(1-e^2)$, or equivalent, to find $e$ or $ae$ for $D$ or $E$.   M1

For $S$:  $a = 5$ and $b = 3$,    $e = \dfrac{4}{5}$,    $ae = 4$    ignore sign with $ae$   A1

For $T$:  $a' = 3$ and $b' = 2$,    $e' = \dfrac{\sqrt{5}}{3}$,    $a'e' = \sqrt{5}$    ignore sign with $a'e'$   A1

$$ST = \sqrt{(16+5)} = \sqrt{21}$$   M1 A1   (5)

[8]

| Question Number | Scheme | Marks |
|---|---|---|
| 3. | $$\dfrac{dy}{dx} = \dfrac{1}{4}\left(4x - \dfrac{1}{x}\right)$$ | B1 |
| | $$\int\left(1+\left(\dfrac{dy}{dx}\right)^{2}\right)^{\!\!\frac{1}{2}}dx = \int\left(1+\left(x-\dfrac{1}{4x}\right)^{2}\right)^{\!\!\frac{1}{2}}dx$$ | M1 |
| | $$=\int\left(1+x^{2}+\dfrac{1}{16x^{2}}-\dfrac{1}{2}\right)^{\!\!\frac{1}{2}}dx = \int\left(\left(x+\dfrac{1}{4x}\right)^{2}\right)^{\!\!\frac{1}{2}}dx = \int\left(x+\dfrac{1}{4x}\right)dx$$ | M1 A1 |
| | $$=\dfrac{x^{2}}{2}+\dfrac{\ln x}{4}$$ | A1 |
| | $$\left[\dfrac{x^{2}}{2}+\dfrac{\ln x}{4}\right]_{0.5}^{2} = 2 + \dfrac{\ln 2}{4} - \dfrac{1}{8} - \dfrac{\ln 0.5}{4} = \dfrac{15}{8} + \dfrac{1}{2}\ln 2$$ | M1 A1   (7) |
| | $$\left(a=\dfrac{15}{8},\ b=\dfrac{1}{2}\right)$$ | [7] |

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** | **(a)** $$\cosh A \cosh B - \sinh A \sinh B = \left(\frac{e^A + e^{-A}}{2}\right)\left(\frac{e^B + e^{-B}}{2}\right) - \left(\frac{e^A - e^{-A}}{2}\right)\left(\frac{e^B - e^{-B}}{2}\right)$$ | M1 |
| | $$= \frac{1}{4}\left(e^{A+B} + e^{-A+B} + e^{A-B} + e^{-A-B} - e^{A+B} + e^{-A+B} + e^{A-B} - e^{-A-B}\right)$$ | |
| | $$= \frac{1}{4}\left(2e^{-A+B} + 2e^{A-B}\right) = \frac{e^{A-B} + e^{-(A-B)}}{2} = \cosh(A-B) \quad \ast \qquad \text{cso}$$ | M1 A1 **(3)** |
| | **(b)** $$\cosh x \cosh 1 - \sinh x \sinh 1 = \sinh x$$ | M1 |
| | $$\cosh x \cosh 1 = \sinh x (1 + \sinh 1) \quad \Rightarrow \quad \tanh x = \frac{\cosh 1}{1 + \sinh 1}$$ | M1 |
| | $$\tanh x = \frac{\dfrac{e + e^{-1}}{2}}{1 + \dfrac{e - e^{-1}}{2}} = \frac{e + e^{-1}}{2 + e - e^{-1}} = \frac{e^2 + 1}{e^2 + 2e - 1} \quad \ast$$ | M1 A1 **(4)** |
| | cso | |
| | | **[7]** |
| | *Alternative for* (b) | |
| | $$\frac{e^{x-1} + e^{-(x-1)}}{2} = \frac{e^x + e^{-x}}{2}$$ | M1 |
| | Leading to $$e^{2x} = \frac{e^2 + e}{e - 1}$$ | M1 |
| | $$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{e^2 + e - (e-1)}{e^2 + e + (e-1)} = \frac{e^2 + 1}{e^2 + 2e - 1} \quad \ast \qquad \text{cso}$$ | M1 A1 **(4)** |

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| 5. | (a) $x = t - \sin 2t \;\Rightarrow\; \dot{x} = 1 - 2\cos 2t \;\Rightarrow\; \ddot{x} = 4\sin 2t$ | either | M1 | |
| | $\quad\;\; y = \cos 2t \;\Rightarrow\; \dot{y} = -2\sin 2t \;\Rightarrow\; \ddot{y} = -4\cos 2t$ | both | A1 | |
| | Obtaining $\dot{x}^2 + \dot{y}^2$ and $\dot{x}\ddot{y} - \ddot{x}\dot{y}$ in terms of $t$ | | M1 | |
| | $\dot{x}^2 + \dot{y}^2 = 1 + 4\cos^2 2t - 4\cos 2t + 4\sin^2 2t = 5 - 4\cos 2t$ | | A1 | |
| | $\dot{x}\ddot{y} - \ddot{x}\dot{y} = -4\cos 2t\left(1 - 2\cos 2t\right) - 4\sin 2t\left(-2\sin 2t\right) = 8 - 4\cos 2t$ | | A1 | |
| | $$\rho = \frac{\left(5 - 4\cos 2t\right)^{3/2}}{8 - 4\cos 2t}$$ | | A1 | (6) |
| | (b) The least value of $y\left(\cos 2t\right)$ is $-1$ | | B1 | |
| | $$\rho = \frac{\left(5 + 4\right)^{3/2}}{8 + 4} = \frac{9}{4}$$ accept equivalent fractions or 2.25 | | B1 | (2) |
| | | | | [8] |

| Question Number | Scheme | Marks |
|---|---|---|
| 6. (a) | $I_n = -\dfrac{3}{4}\left[x^n(8-x)^{4/3}\right]_0^8 + \dfrac{3}{4}\int nx^{n-1}(8-x)^{4/3}\,dx$ | M1 A1 |
| | $= \dfrac{3}{4}\int nx^{n-1}(8-x)^{4/3}\,dx$       ft numeric constants only | A1ft |
| | $\int nx^{n-1}(8-x)(8-x)^{1/3}\,dx = \int nx^{n-1}8(8-x)^{1/3}\,dx - \int nx^{n-1}x(8-x)^{1/3}\,dx$ | M1 A1 |
| | $I_n = 6nI_{n-1} - \dfrac{3}{4}nI_n \;\Rightarrow\; I_n = \dfrac{24n}{3n+4}I_{n-1}$   **✱**      cso | A1     (6) |
| (b) | $I_0 = \displaystyle\int_0^8 (8-x)^{1/3}\,dx = \left[-\dfrac{3}{4}(8-x)^{4/3}\right]_0^8 = \dfrac{3}{4}\times 8^{4/3} = 12$ | M1 A1 |
| | $I = \displaystyle\int_0^8 x(x+5)(8-x)^{1/3}\,dx = I_2 + 5I_1$ | M1 |
| | $I_1 = \dfrac{24}{7}I_0, \quad I_2 = \dfrac{48}{10}I_1 = \dfrac{48}{10}\times\dfrac{24}{7}I_0 \left(=\dfrac{576}{35}I_0\right)$ | M1 A1 |
| | $\left(\text{The previous line can be implied by } I = I_2 + 5I_1 = \dfrac{168}{5}I_0\right)$ | |
| | $I = \left(\dfrac{576}{35} + 5\times\dfrac{24}{7}\right)\times 12 = \dfrac{2016}{5}\,(=403.2)$ | A1     (6) |
| | | [12] |

| Question Number | Scheme | Marks |
|---|---|---|
| 7. | (a) $\quad \dfrac{d}{dx}\left(\operatorname{arsinh} x^{\frac{1}{2}}\right)=\dfrac{1}{\sqrt{(1+x)}}\times\dfrac{1}{2}x^{-\frac{1}{2}}\left(=\dfrac{1}{2\sqrt{x}\sqrt{(1+x)}}\right)$ | M1 A1 |

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At $x=4$, $\quad \dfrac{dy}{dx}=\dfrac{1}{4\sqrt{5}}$ $\qquad\qquad$ accept equivalents $\qquad$ A1 $\quad$ (3)

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(b) $\qquad\qquad x=\sinh^2\theta,\qquad\qquad \dfrac{dx}{d\theta}=2\sinh\theta\cosh\theta$

$$\int \operatorname{arsinh}\sqrt{x}\,dx=\int \theta\times 2\sinh\theta\cosh\theta\,d\theta \qquad \text{M1 A1}$$

$$=\int \theta\sinh 2\theta\,d\theta=\dfrac{\theta\cosh 2\theta}{2}-\int\dfrac{\cosh 2\theta}{2}\,d\theta \qquad \text{M1 A1 + A1}$$

$$=\quad\dots\quad -\dfrac{\sinh 2\theta}{4} \qquad \text{M1}$$

$$\left[\dfrac{\theta\cosh 2\theta}{2}-\dfrac{\sinh 2\theta}{4}\right]_0^{\operatorname{arsinh}2}=\quad\dots \qquad \text{attempt at substitution} \qquad \text{M1}$$

$$=\left[\dfrac{\theta(1+2\sinh^2\theta)}{2}-\dfrac{2\sinh\theta\cosh\theta}{4}\right]=\dfrac{1}{2}\operatorname{arsinh}2\times(1+8)-\dfrac{4\sqrt{5}}{4} \qquad \text{M1 A1}$$

$$=\dfrac{9}{2}\ln\left(2+\sqrt{5}\right)-\sqrt{5} \qquad \text{A1} \quad (10)$$

$$[13]$$

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*Alternative for* (a)

$$x=\sinh^2 y,\qquad 2\sinh y\cosh y\dfrac{dy}{dx}=1 \qquad \text{M1}$$

$$\dfrac{dy}{dx}=\dfrac{1}{2\sinh y\cosh y}=\dfrac{1}{2\sinh y\sqrt{(\sinh^2 y+1)}}\left(=\dfrac{1}{2\sqrt{x}\sqrt{(1+x)}}\right) \qquad \text{A1}$$

At $x=4$, $\quad \dfrac{dy}{dx}=\dfrac{1}{4\sqrt{5}}$ $\qquad$ accept equivalents $\qquad$ A1 $\quad$ (3)

*An alternative for* (b) *is given on the next page*

| Question Number | Scheme | Marks |
|---|---|---|
| 7. | *Alternative for* (b) | |

$$\int 1 \times \operatorname{arsinh}\sqrt{x}\,dx = x\operatorname{arsinh}\sqrt{x} - \int x \times \frac{1}{2\sqrt{x}\sqrt{(1+x)}}\,dx$$    M1 A1 + A1

$$= x\operatorname{arsinh}\sqrt{x} - \int \frac{\sqrt{x}}{2\sqrt{(1+x)}}\,dx$$

Let $x = \sinh^2\theta$, $\dfrac{dx}{d\theta} = 2\sinh\theta\cosh\theta$

$$\int \frac{\sqrt{x}}{\sqrt{(1+x)}}\,dx = \int \frac{\sinh\theta}{\cosh\theta} \times 2\sinh\theta\cosh\theta\,d\theta$$    M1 A1

$$= 2\int \sinh^2\theta\,d\theta = 2\int \frac{\cosh 2\theta - 1}{2}\,d\theta, \quad = \frac{\sinh 2\theta}{2} - \theta$$    M1, M1

$$\left[\frac{\sinh 2\theta}{2} - \theta\right]_0^{\operatorname{arsinh}2} = \left[\frac{2\sinh\theta\cosh\theta}{2} - \theta\right]_0^{\operatorname{arsinh}2} = \frac{2\times 2\times\sqrt{5}}{2} - \operatorname{arsinh}2$$    M1 A1

$$\int_0^4 \operatorname{arsinh}\sqrt{x}\,dx = 4\operatorname{arsinh}2 - \frac{1}{2}\left(\frac{2\times 2\times\sqrt{5}}{2} - \operatorname{arsinh}2\right) = \frac{9}{2}\ln\left(2+\sqrt{5}\right) - \sqrt{5}$$    A1    **(10)**

*The last 7 marks of the alternative solution can be gained as follows*

Let $x = \tan^2\theta$, $\dfrac{dx}{d\theta} = 2\tan\theta\sec^2\theta$

$$\int \frac{\sqrt{x}}{\sqrt{(1+x)}}\,dx = \int \frac{\tan\theta}{\sec\theta} \times 2\tan\theta\sec^2\theta\,d\theta$$    dependent on first M1    M1 A1

$$= \int 2\sec\theta\tan^2\theta\,d\theta$$

$$\int(\sec\theta\tan\theta)\tan\theta\,d\theta = \sec\theta\tan\theta - \int \sec^3\theta\,d\theta$$    M1

$$= \sec\theta\tan\theta - \int \sec\theta(1+\tan^2\theta)\,d\theta$$

Hence $\displaystyle\int \sec\theta\tan^2\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\int \sec\theta\,d\theta$

$$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln(\sec\theta + \tan\theta)$$    M1

$$\left[\ \ldots\ \right]_0^{\arctan 2} = \frac{1}{2}\times\sqrt{5}\times 2 - \frac{1}{2}\ln\left(\sqrt{5}+2\right)$$    M1 A1

$$\int_0^4 \operatorname{arsinh}\sqrt{x}\,dx = 4\operatorname{arsinh}2 - \sqrt{5} + \frac{1}{2}\ln\left(2+\sqrt{5}\right) = \frac{9}{2}\ln\left(2+\sqrt{5}\right) - \sqrt{5}$$    A1

| Question Number | Scheme | Marks |
|---|---|---|
| 8. | **(a)** Gradient of $PQ = \dfrac{2ap - 2aq}{ap^2 - aq^2} = \dfrac{2}{p+q}$  Can be implied | B1 |
| | Use of any correct method or formula to obtain an equation of $PQ$ in any form. | M1 |
| | Leading to $(p+q)y = 2(x + apq)$ **\*** | A1 (3) |
| | **(b)** Gradient of normal at $P$ is $-p$.  Given or implied at any stage | B1 |
| | Obtaining any correct form for normal at either point. Allow if just written down. | M1 A1 |
| | $y + px = 2ap + ap^3$ | |
| | $y + qx = 2aq + aq^3$ | |
| | Using both normal equations and eliminating $x$ or $y$. Allow in any unsimplified form. | M1 |
| | $(p-q)x = 2a(p-q) + a(p^3 - q^3)$  Any correct form for $x$ or $y$ | A1 |
| | Leading to $x = a(p^2 + q^2 + pq + 2)$ **\***  cso | A1 |
| | $y = -apq(p+q)$ **\***  cso | A1 (7) |
| | **(c)** $0 = 2(5a + apq) \Rightarrow pq = -5$ | B1 |
| | Using $pq = -5$ in both $x = a(p^2 + q^2 + pq + 2)$ and $y = -apq(p+q)$. | M1 |
| | $x = a(p^2 + q^2 - 3)$  $y = 5a(p+q)$ | |
| | Any complete method for relating $x$ and $y$, independently of $p$ and $q$. | M1 |
| | A correct equation in any form. | A1 |
| | $x = a\left((p+q)^2 - 2pq - 3\right) = a\left(\left(\dfrac{y}{5a}\right)^2 + 10 - 3\right)$ | |
| | Leading to $y^2 = 25a(x - 7a)$  Accept equivalent forms of $f(x)$ | A1 (5) |
| | | [15] |
| | The algebra above can be written in many alternative equivalent forms. | |